Laplacian Growth, Nonequilibrium Patterns and Integrable Systems

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Laplacian growth is an evolution of a plane interface between two immiscible phases, driven by a gradient of a harmonic field. An interface typically develops a complex unstable fingering patterns. These kind of patterns occur in vast number of phenomena ranging from astrophysics to semiconductor nanostructures. Regardless of details of a system and initial conditions these patterns have some general features that are captured by a number of simple models. The most famous of these is diffusion-limited aggregation.

The patterns reveal scaling properties of nonequilibrium statistical physics, which continue to remain a major unsolved problem. Progress in understanding fingering patterns has been driven by recently recognized connections between this class of phenomena and two branches of mathematical physics: integrable (soliton) equations, and random matrix theory.

Laplacian Growth and Witham equations of Soliton Theory: I. Krichever, M. Mineev-Weinstein, P. Wiegmann, A. Zabrodin, nlin.SI/0311005

A grown fingering pattern: Air inserted to the cell filled by silicon oil (H. Swinney)

DLA - simple computer simulation of growing fractal cluster. What are its scaling properties?